The CRISP Consortium

# Pore Pressure Analysis

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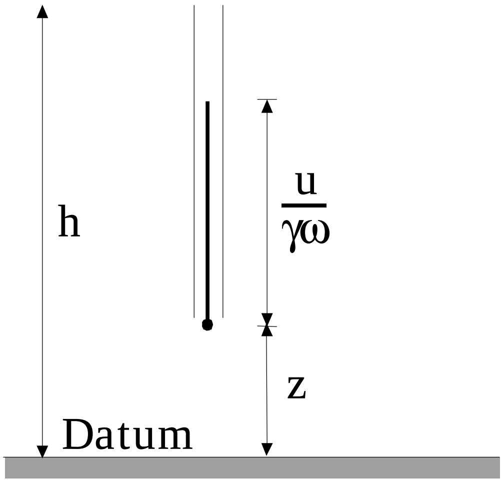
Produced by the CRISP Consortium Ltd

## DEFINITION OF EXCESS PORE PRESSURE USED IN CRISP

The pore pressure at A is u. If a standpipe is inserted at A the water rises to a height $\frac{u}{gw}$.




**Using the normal soil mechanics convention**

Total head = pressure head + elevation head

$$h = \frac{u}{gw} + z$$

It is spatial variation of h which causes water to flow though the ground in transient or steady seepage.

In CRISP the excess pore pressure, $\bar{u}$, is defined as ;

$$\bar{u} = hgw$$

## CONSOLIDATION EQUATIONS (BIOT)

**a) Water is flowing though an element of soil at a rate controlled by Darcy's Law ;**

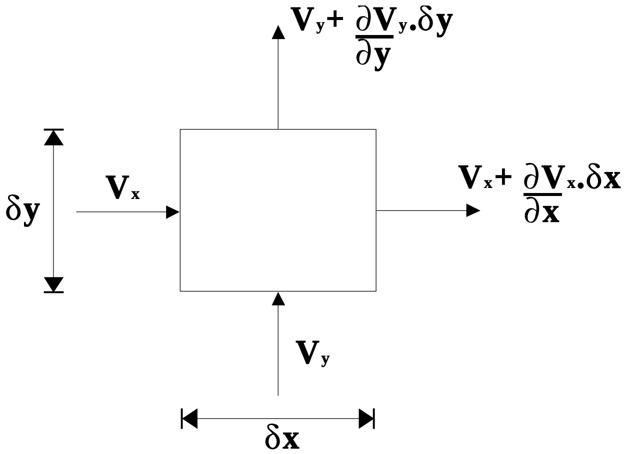
$$V_x = -k_x \frac{\partial h}{\partial x} = -\frac{k_x}{gw}\frac{d\bar{u}}{\partial x}$$

$$V_y = -k_y \frac{\partial h}{\partial y} = -\frac{k_y}{gw}\frac{\partial \bar{u}}{\partial y}$$

$k_x,k_y$ = permeability's in x and y directions
$V_x,V_y$= artificial seepage velocities in x and y directions

## CONSOLIDATION EQUATIONS - continued

### b) There is continuity of flow of water (grains assumed incompressible)




$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} = \frac{d(e_r)}{dt}$$

$\varepsilon_v$ = volumetric strain
(compression positive)
t = time

### c) Principle of effective stress

$$s_x = s_x' + u$$

$$s_y = s_y' + u$$

Also the incremental form

$$\Delta s_x = \Delta s_x' + \Delta u$$

$$\Delta s_y = \Delta s_y' + \Delta u$$

Referring back to the definition of u and excluding large movements

$$\Delta \bar{u} = \Delta u$$

**d) Differential equations of equilibrium**

$$\frac{\partial s_x}{\partial x} + \frac{\partial t_{xy}}{\partial y} = w_x$$

$$\frac{\partial t_{xy}}{\partial x} + \frac{\partial s_y}{\partial y} = w_y$$

**e) Stress - strain**

$$\underline{s}' = D\underline{e} \qquad\qquad \underline{s} = \begin{bmatrix} s_x \\ s_y \\ t_{xy} \end{bmatrix} \quad e = \begin{bmatrix} e_x \\ e_y \\ g_{xy} \end{bmatrix}$$

D is the matrix of elastic constants <u>or</u>

$$\Delta\underline{s}' = D\Delta\underline{e}$$

(i.e. any stress-strain law in incremental form)

Equations a) and e) are combined and two sets of partial differential equations are obtained, one set describing equilibrium and the other set describing continuity of water flow.
The integral form of these equations is obtained using virtual work or Galerkin's weighted residual technique (see Britto & Gunn for details)
A finite element discretisation is introduced

$$\underline{d} = N\underline{a}$$

$$\bar{\underline{u}} = \bar{N}\underline{b}$$

where <u>a</u> = nodal displacements
<u>b</u> = nodal excess pore pressures

We now make the appropriate substitutions in the integral equations and integrate the continuity equations with respect to time to obtain

$$\begin{bmatrix} K & L \\ L^T & -\Phi\Delta t \end{bmatrix} \begin{bmatrix} \Delta\underline{a} \\ \Delta\underline{b} \end{bmatrix} = \begin{bmatrix} \Delta\underline{r}_1 \\ \Delta\underline{r}_2 \end{bmatrix}$$

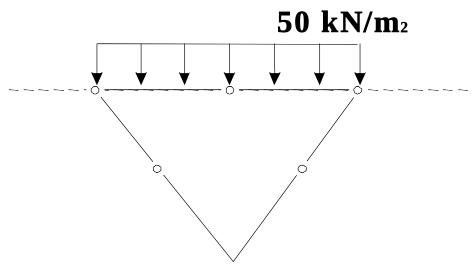
Note that these equations are incremented form, describing a change in <u>a</u> and <u>b</u> to <u>a</u> + Δa, a + Δ<u>u</u> over a time interval Δt.
Thus consolidation analysis in CRISP requires the division of the total time into a number of increments.

# PORE PRESSURE BOUNDARY CONDITIONS

Note that all boundary conditions in CRISP are applied in an incremental fashion. (except at the insitu stage)

Suppose an incremental pressure loading of 5kN/m² is applied for each of the first 10 increments of an analysis.
Then at the end of 10 increments, a total load of 50 kN/m² will act. If no additional loads are applied in the increments that follow, this total load of 50 will remain.

Displacement boundary conditions ("fixities") are dealt with in the same way.

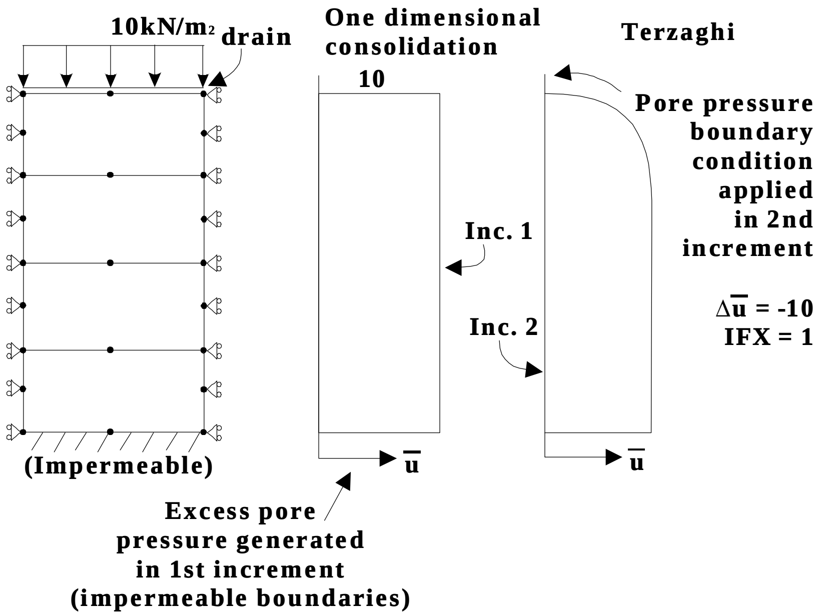



Pore pressure boundary conditions are dealt with under the same heading as displacements (record M).

In typical analysis some external loads are applied to the mesh and this generates excess pore pressures. These pore pressures then dissipate over a period of time.

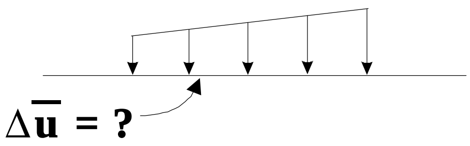
Experience has shown that it is better to apply the external loads whilst keeping the boundaries impermeable. Pore pressure boundary conditions are then "switched on" in the increment following the last increment applying external loads.
This procedure avoids problems which sometimes occur with the simultaneous application of both types of boundary condition (oscillation of pore pressure).

## Example

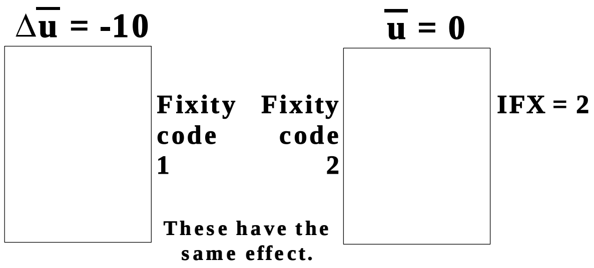


## PORE PRESSURE FIXITY CODE 2

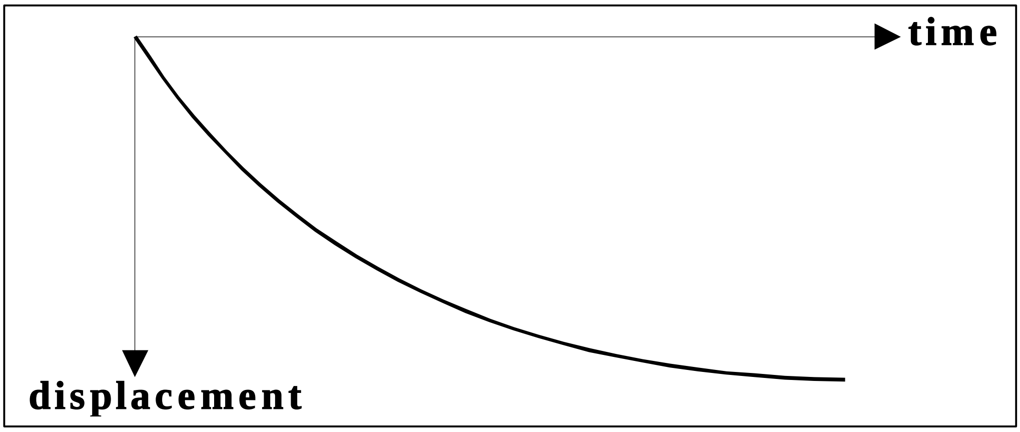
In the general case we don't know what the excess pore pressure generated on the boundary will be.
For this reason pore pressure fixity code 2 is provided.
This allows the user to prescribe an <u>absolute</u> ( as opposed to incremental) excess pore pressure on the boundary.




Terzaghi example again



# SELECTION OF TIMESTEPS




For an efficient solution we advise that time steps should be selected on an approximately logarithmic basis with respect to time.
For example :

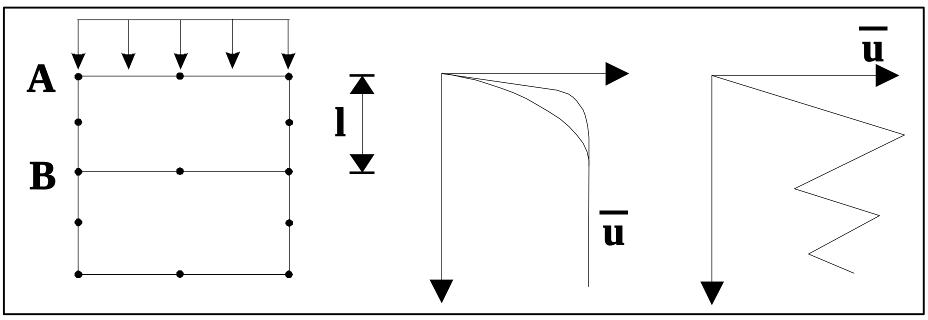
| Inc. no. | DTIME | TTIME |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 1 | 2 |
| 3 | 3 | 5 |
| 4 | 5 | 10 |
| 5 | 10 | 20 |
| 6 | 30 | 50 |
| 7 | 50 | 100 |
| 8 | 100 | 200 |
| 9 | 300 | 500 |
| 10 | 100 | 1000 |

The above scheme will be modified ;

a) in a non-linear analysis the stress increment should be restricted to a reasonable size ( more time step / increments)

b) Time steps which are too short can lead to numerical problems, pore pressure oscillations etc.

This arises particularly in the case when the pore pressure boundary condition is introduced



## SELECTION OF TIME STEPS (continued)

Here the time step should be large enough so that the pore pressure modes near the boundary (such as b) just feel the effect of the drainage boundary.

The following equation, based on an advanced parabolic isochrone can be used to estimate this time :

$$t = \frac{l^2}{12C_v}$$

Where $C_v$ has its normal soil mechanics meaning

$$C_v = \frac{k}{m_v \mathit{gw}} = \frac{kE_o'}{\mathit{gw}} = \frac{kE'(1-n')}{(1+n')(1-2n')\mathit{gw}}$$

l = width of element next to boundary
In a non linear analysis there is often a conflict arising between a) and b) above. This is usually dealt with by applying the initial loading using short time steps and an impermeable boundary. The drainage boundary is the “switched on” with a sufficiently large time step.

In general, problems with time steps can be over come by increasing (or less commonly, decreasing) the time step by a factor of 10 and trying again.

## MORE ON PORE PRESSURE BOUNDARY CONDITIONS

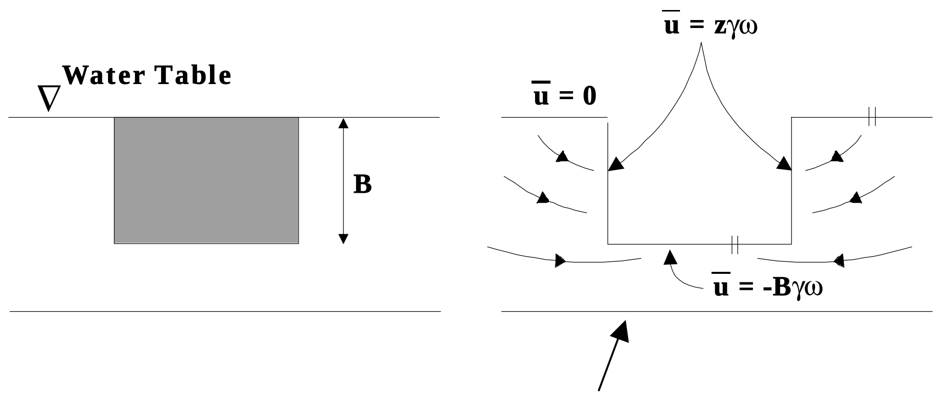
Any boundary where there is no pore pressure boundary condition applied is assumed to be impermeable. If no pore pressure boundary conditions are applied then the excess pore pressures will end up at some constant value depending on the loading and geometry.

Sometimes it is necessary to fix excess pore pressures on internal boundaries;

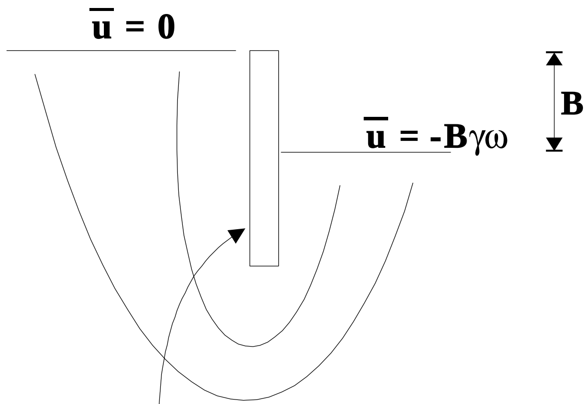
Fix u onboundaries

| | |
|---|---|
| Sand (drained) | Type 4 |
| Clay (consolidate) | Type 5 |
| Sand (drained) | Type 4 |

## PORE PRESSURE BOUNDARY CONDITIONS FOR EXCAVATIONS



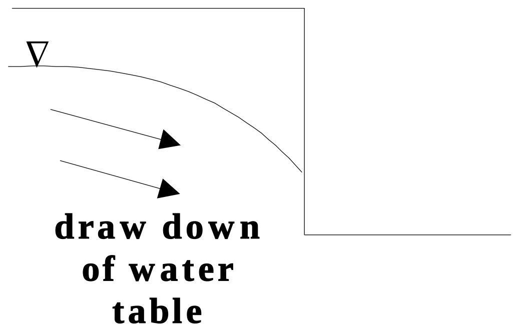

assumes free water is available at the bottom and on the sides of the excavation (initially water maybe sucked in)



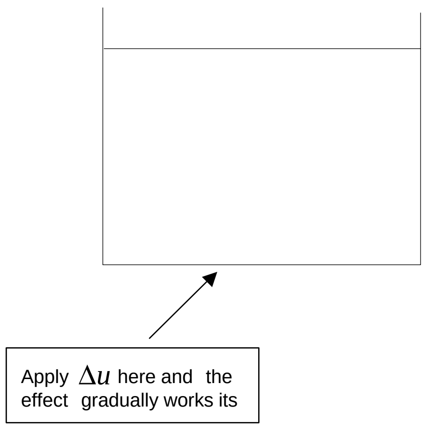

**Retaining wall elements are type 4 and this will be taken as an impermeable boundary.**

## **PORE PRESSURE BOUNDARY CONDITIONS - SOME FINAL POINTS**

i) CRISP can't model unconfined flow problems - where the phreatic surface ends up horizontal




ii) On the other hand a rising water table can be modelled




iii) Adding consolidation elements to a mesh is possible (in principle) but probably unwise (added elements are assumed to have zero insitu stresses).

## EFFECT OF SEEPAGE FORCES

If the final state is one of steady seepage CRISP automatically takes account of deformations due to seepage forces.

Consider, for example, a retaining wall analysis.

The undrained analysis can be done in three different ways;
Total stress (e.g. $E_u$, $\nu_u$)
Effective stress ( E', ν', $K_\omega$)
Consolidation with a short time step (E', ν')

CRISP will give the same answers for a), b), and c) (if equivalent properties have been assumed).

Drained (long term) analysis can be done in two different ways;

Effective stress ( E', ν')
Consolidation ( E', ν') with several time steps

The results from a) and b) will be different - only b) includes the seepage forces which occur when there is under drainage.

For a simple example see section 3.6.4 of Britto & Gunn. Soil deformations are being caused by changes in the pore pressure boundary conditions, not external loads.

## Example

The following figures are taken from an article in the Canadian Geotechnical Journal by Almeida, Britto & Parry (1986, 23 ,103-114).

CRISP analysis are compared with a model embankment tested on the Cambridge Geotechnical centrifuge.

The embankment was constructed in stages “in flight” (i.e. without stopping the centrifuge).

Modified Cam-clay was used to model the clay foundation.

Note in particular the good “predictions” of pore pressure and the yielding/strengthening of the clay.