The CRISP Consortium

Critical State Soil Mechanics

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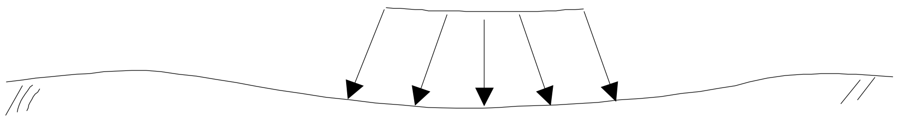
Produced by the CRISP Consortium Ltd

SOIL MODELLING

"model" $\equiv$ assumed relationship between stress and strain for a soil.



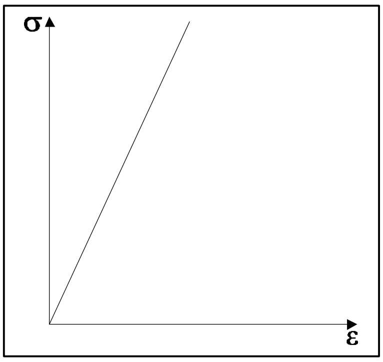
Underlying conventional design calculations in geotechnical engineering are different soil models based on concepts of elasticity and plasticity.

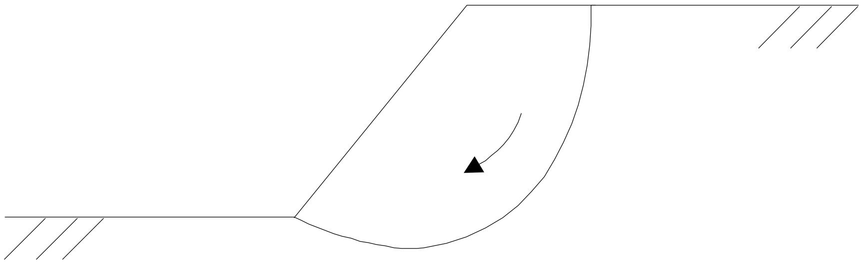
Underlying most methods of calculating ground movements is the assumption of a linear elastic soil model.

E = Young's modulus
ν = Poisson's ratio

<u>Drained</u>
E', ν'

<u>Undrained</u>
E_u, ν_u



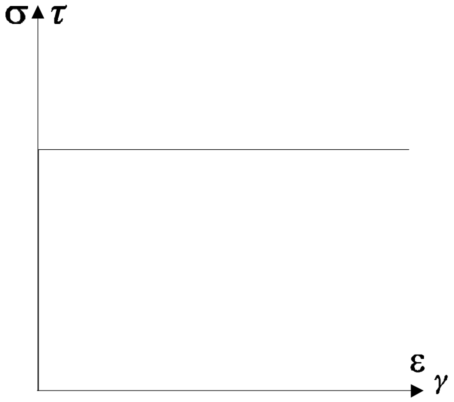


Underlying most stability calculations is a soil model which assumes rigid, perfectly plastic behaviour.

Strength parameters:

<u>Drained</u>
c', ϕ'

<u>Undrained</u>
c_u



CRITICAL STATE SOIL MECHANICS

CSSM provides soil models which include:

- elastic strains and plastic yielding before failure
- dilatancy (volumetric contraction or expansion on shearing)
- existence of critical states
- provides soil models which can be used as the basis of numerical predictions (using finite elements)
- provides the basis for reviewing data from soil tests and selecting strength and stiffness parameters for design

CRITICAL STATE PARAMETERS

3 parameters are used to describe the (changing) state of a soil sample in a triaxial test.
These are :

- Effective mean stress,
- Deviatoric stress
- Specific volume

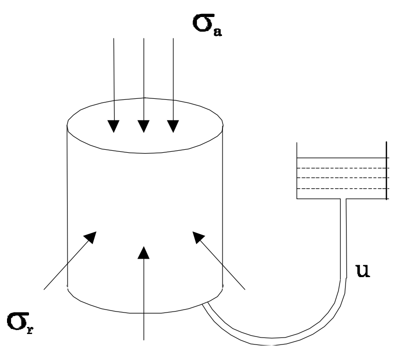
$$\boxed{p', q, V}$$

<u>Total stresses</u>

σ_a, σ_r

<u>Effective stresses</u>

$\sigma_a' = \sigma_a - u$
$\sigma_r' = \sigma_r - u$



$$\boxed{p' = \frac{\sigma_a + 2\sigma_r'}{3}} = \frac{\sigma_a + 2\sigma_r}{3} - u$$

$= p - u$

$$\boxed{q = \sigma_a - \sigma_r} = \sigma_a' - \sigma_r'$$

V - the specific volume = 1 + e

Relationship Between Specific Volume and Other Measures of Soil Density

	Volumes	Ratio of volumes	Weights	Ratio of weights
Water	V_w	e	$V_w\gamma_w$	w
Solid	V_s	1	$V_s G_s \gamma_w$	1

e	is void ratio
w	is moisture content
γ_w	is unit weight of water
G_s	is specific gravity of solid phase

V = 1 + e = specific volume = volume containing unit volume of solid material.

Now $$e = \frac{V_w}{V_s}$$

and $$w = \frac{V_w\,\gamma_w}{V_s\,G_s\,\gamma_w} = \frac{e}{G_s}$$

So e = G_s. w

and V = 1 + e = 1 + G_s w

bulk unit weight of soil, γ

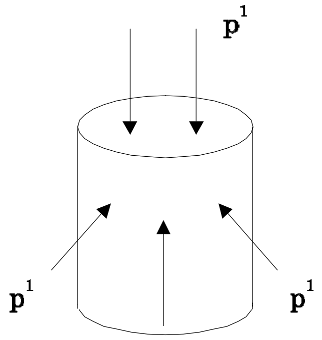
$$= \frac{V_w\,\gamma_w + V_s\,G_s\,\gamma_w}{V_w + V_s} = \frac{e}{G_s}$$

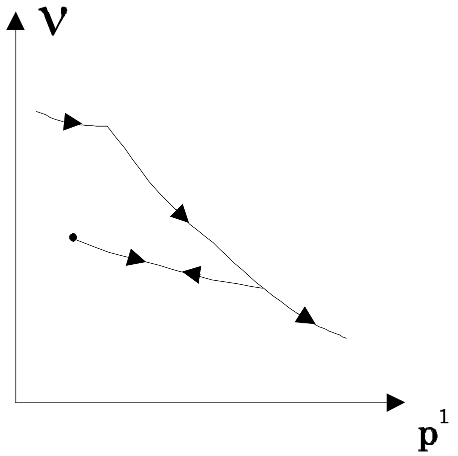
$$\boxed{\gamma = \frac{(e + G_s)\gamma_w}{e + 1}}$$

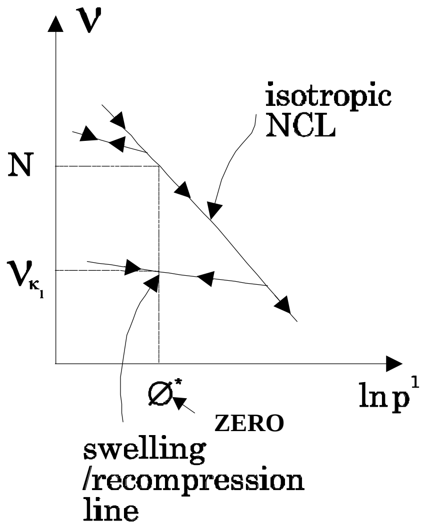
(often called bulk density, although units are force / volume, not mass / volume.)

Observed Volume - Pressure Relations

(isotropic compression)







$$\boxed{V = N - \lambda \ln p'}$$ isotropic NCL

$$\boxed{V = V_\kappa - \kappa \ln p'}$$ swelling and recompression

THE CRITICAL STATE LINE (CSL)

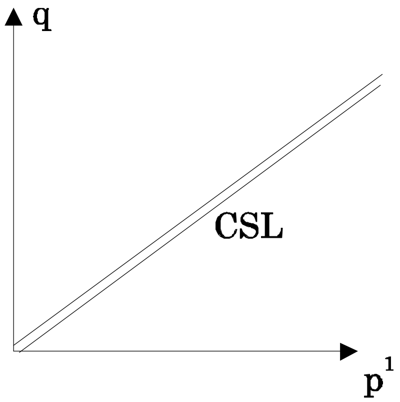
If a soil is continually sheared then it will eventually reach a critical state in which further shear strains can occur with no changes in effective stresses or volume.

When a soil is at the critical state:

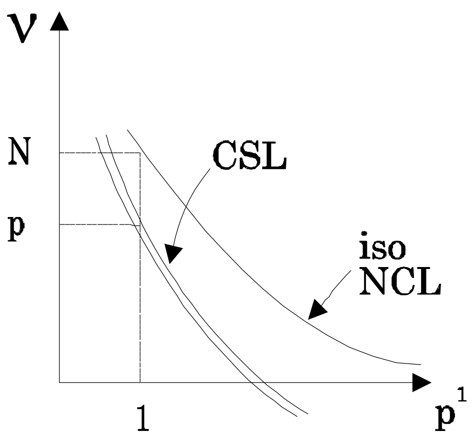
$$\boxed{q = Mp'}$$

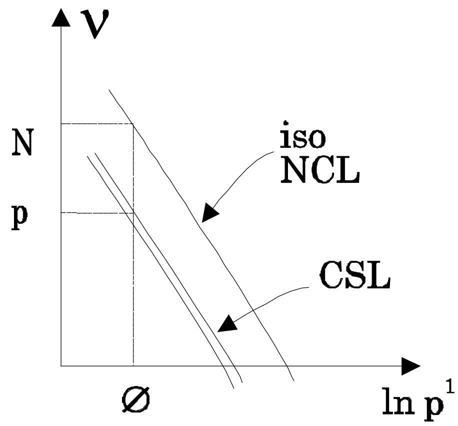
$$\boxed{V = \Gamma - \lambda \ln p'}$$

M and Γ are constants for a particular soil.



Conventionally the CSL is shown as a pair of lines (really it is just one line).





<u>Summary</u>

M, N, Γ, κ, λ are soil constants

p', q, V (and V_κ) vary during a test.

Prediction of Final States of Triaxial Tests using Critical State Theory

(a) establish starting point in (p', q) and (p', v) diagrams

(b) establish test path in one of these diagrams:

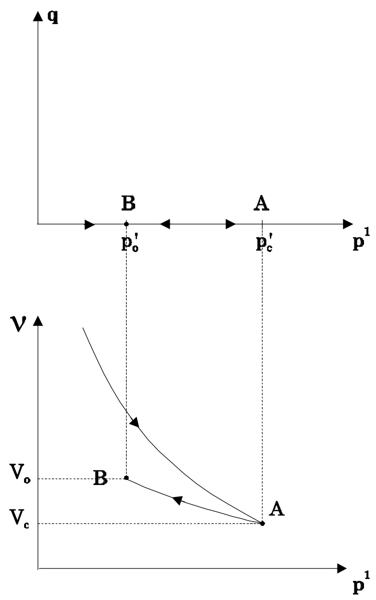
- (p', q) for drained test
- (p', v) for undrained test

(c) calculate intersection point of test path and critical state line

→ failure condition

(a) STARTING POINTS

An over-consolidated sample is prepared by drained isotropic compression to point A and then drained unloading to point B.



$V_c \quad = \quad N - \lambda \ln p_c'$ (1)

$V_\kappa \quad = \quad V_c + \kappa \ln p_c'$ (2)

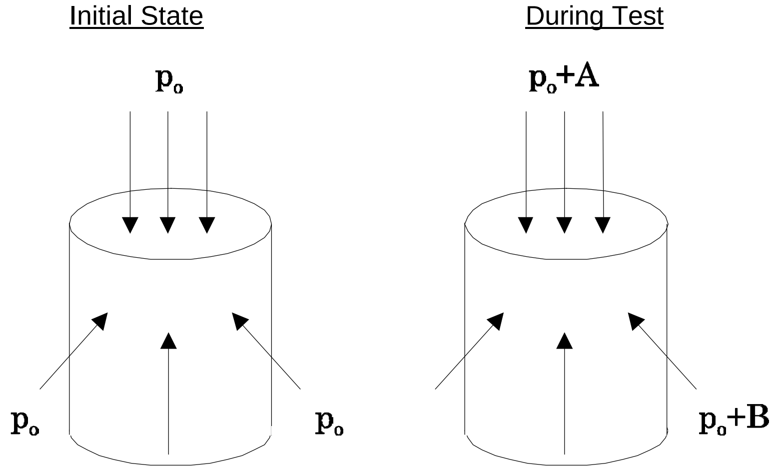
$V_\kappa \quad = \quad V_o + \kappa \ln p_o'$ (3)

Eliminate V_κ, V_c from (1), (2), (3):

$$\boxed{V_o = N - \lambda \ln p_c' + \kappa \ln \left(\frac{p_c'}{p_o'}\right)}$$

(b) TEST PATHS

The total stress path is always controlled by the way in which the soil sample is loaded.



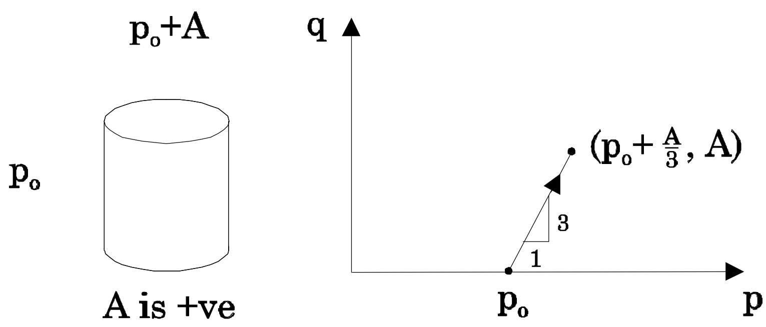
$p = p_o$ $\qquad$ $p = p_o + \dfrac{A + 2B}{3}$

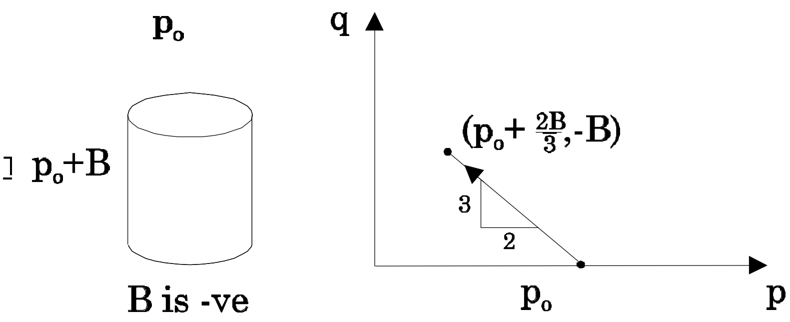
$q = 0$ $\qquad$ $q = A - B$

A and B have differing relative magnitudes according to type of test.

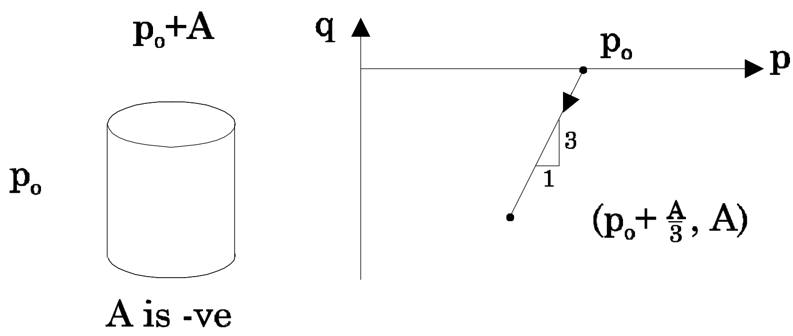
TEST PATHS (contd.)

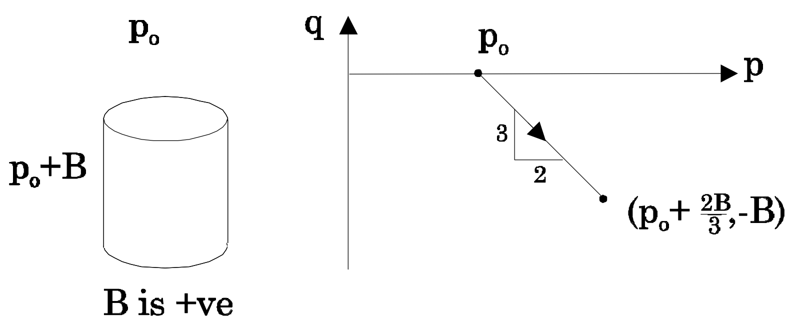
Compression Tests



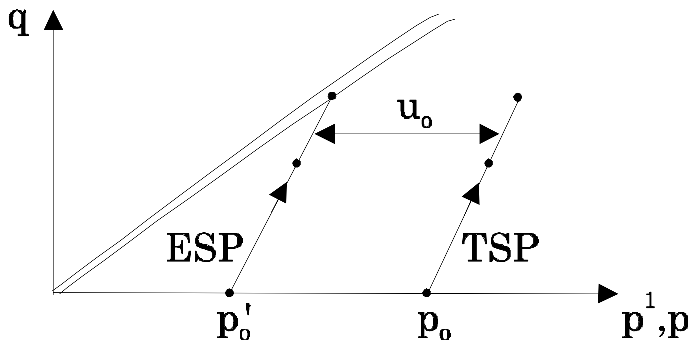


Extension Tests





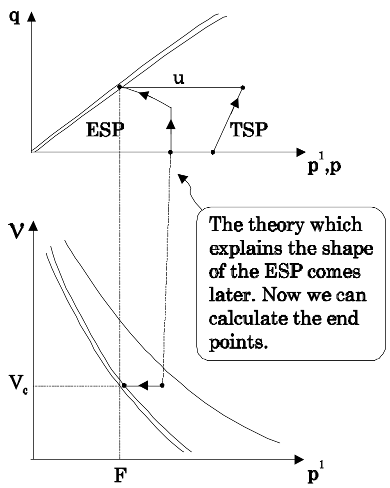
The pore pressure is zero (i.e. atmospheric pressure) or held at a constant “back pressure”, say u_o.



So the effective stress path (ESP) is parallel to the total stress path (TSP), or coincident with (when $u_o = 0$).

TEST PATH IN AN UNDRAINED TEST

In an undrained test there is no change in volume, so the test path is horizontal in the (p’, V) diagram.



WORKED EXAMPLE

Two identical samples of clay are isotropically normally compressed to an all round effective pressure of 100 kPa and are then allowed to swell back to an effective isotropic pressure of 50 kPa.

The first sample is then subjected to a standard drained compression test. What is the deviator stress at failure and what is the volumetric strain experienced by the sample at failure?

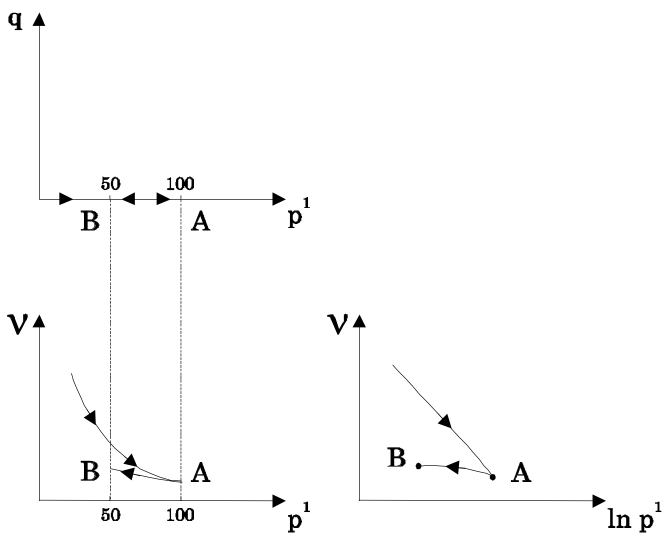
The second sample is subjected to a standard undrained compression test.

What are the deviator stress and pore pressure at failure, if there is initially a back pressure of 50 kPa?

Assume that the soil has the following critical state properties:

$M = 0.95$, $\lambda = 0.093$, $\kappa = 0.035$, $\Gamma = 2.06$ and $N = 2.118$

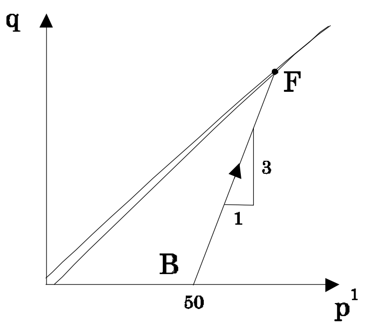
(a) Sample preparation



V_A	=	$N - \lambda \ln 100$	(1)
V_κ	=	$V_A + \kappa \ln 700$	(2)
V_κ	=	$V_B + \kappa \ln 50$	(3)

V_B = 2.118 -0.093 ln 100 + κ ln 2
(N) (λ) (0.035)

(b) Drained test



Drained effective stress path (ESP)

is	$q = -150 + 3p'$	(4)
CSL	$q = 0.95\,p'$	(5)

At end of test (point F):

(4) - (5) $p_F' = \dfrac{150}{2.05} = 73.2$

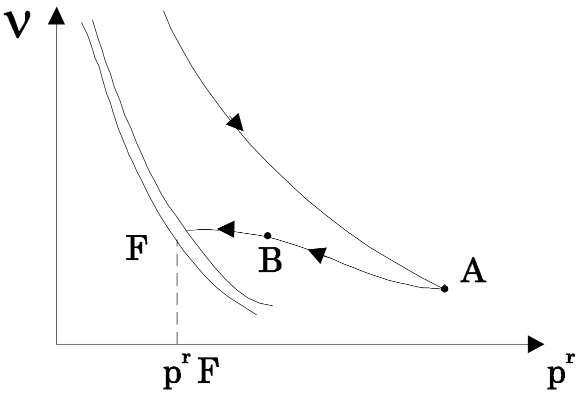
$\underline{\underline{q_F}}$ = 0.95 x 73.2 = $\underline{\underline{69.5}}$ kPa (deviator stress)

V_F	=	$\Gamma - \lambda \ln 73.2$
	=	2.06 - 0.93 ln 73.2
	=	1.661

Vol strain = $\frac{1.714-1661}{1.714}=\underline{\underline{3.1\%}}$

(c) Undrained test

V_B = 1.714 (same as drained) at start.

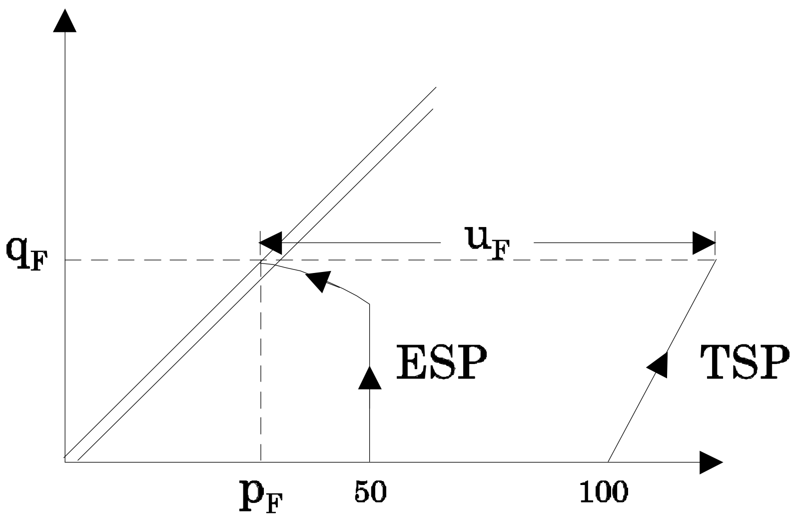


Undrained

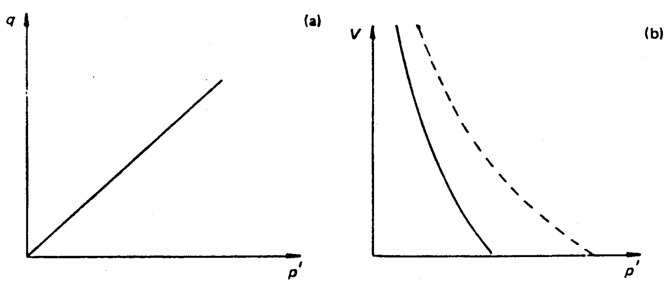
so $V_F = V_B$ also $V_F = \Gamma - \lambda \ln p'_f$

$1.714 = 2.06 - 0.093 \ln p_F$

$$\Rightarrow \ln p_F = \left(\frac{2.06 - 1.714}{0.093}\right)$$

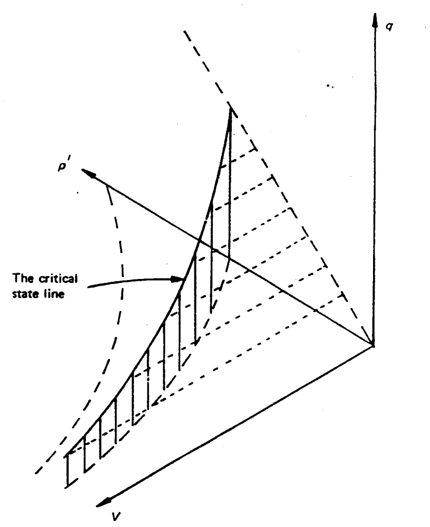


From Britto and Gunn (1987)



The critical state line in (a) (p’, q) plot and (b) (p’, V) plot (isotropic normal compression line is shown dashed in (b)).

$$\frac{dn}{de} = 0; \frac{dq}{de} = 0; \frac{dp'}{de} = 0$$



The critical state line in (p’, V, q) space is given by the intersection of two planes: q = Mp’ and a curved vertical plane V = Γ - λ ln (p’).