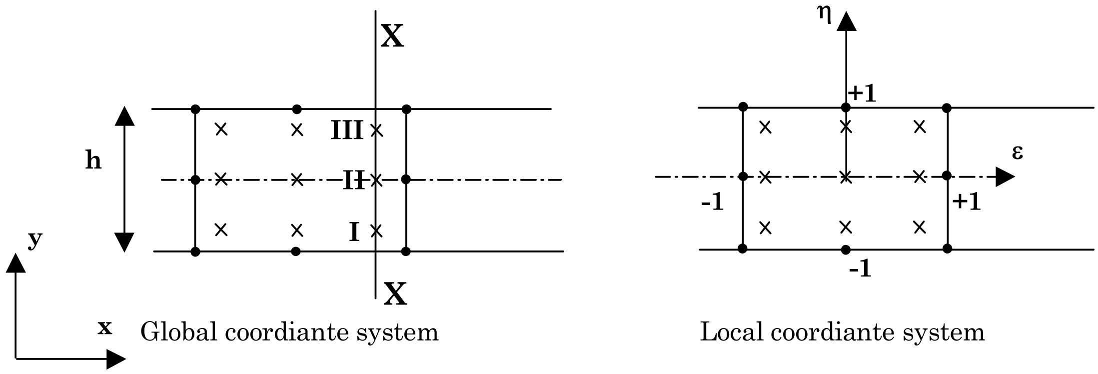
Calculating axial force for 2D quadrilateral elements in CRISP

It is possible to obtain axial from 2D linear strain quadrilateral elements. For a plane strain quadrilateral element representing part of a structure, the axial force across section x-x in Figure 1 is found from the integral:

$$P_a = \int_{-h/2}^{h/2} \sigma_x . dy$$

Where h is the depth of the beam and σ_x is the stress normal to the cross section.



For reasons associated with numerical integration, it is convenient to work with a 'normalised' local coordinate system based on the following relationship

$$\frac{dy}{d\eta} = \frac{h}{2}$$

The relationship between global and local coordinates is given by

$$\frac{\partial y}{\partial \eta} = \frac{h}{2}$$

This leads to $dy = \frac{h}{2} d\eta$

Using the local co-ordinates, we obtain

$$P_a = \left(\frac{h}{2}\right) \int_{-1}^{+1} \sigma_x d\eta .$$

The above integral can be evaluated using a three-point-one- dimensional Gauss integration rule. Thus:

$$P_a = \left(\frac{h}{2}\right) \sum_{i=1}^{3} \sigma_{xi} \times \omega_i$$

The co-ordinates and weights for the three Gauss points are:

Gauss point	η Coordinate	Weight
I	$-\sqrt{3/5}$	5/9
II	0	8/9
III	$\sqrt{3/5}$	5/9

Substituting in the above integral, we obtain:

$$P_a = \frac{h}{2}\left[\sigma_{x.I} \times \frac{5}{9} + \sigma_{x.II} \times \frac{8}{9} + \sigma_{x.III} \times \frac{5}{9}\right]$$

The user would need to find the effective horizontal stresses σ'_x for horizontal members, (or σ'_y for vertical members), then use the above equation to find the axial force.