Modelling plastic anisotropy and destructuration –

S-CLAY1 and S-CLAY1S models

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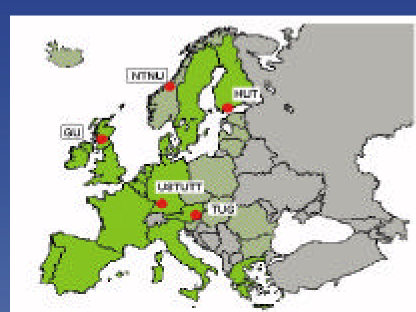
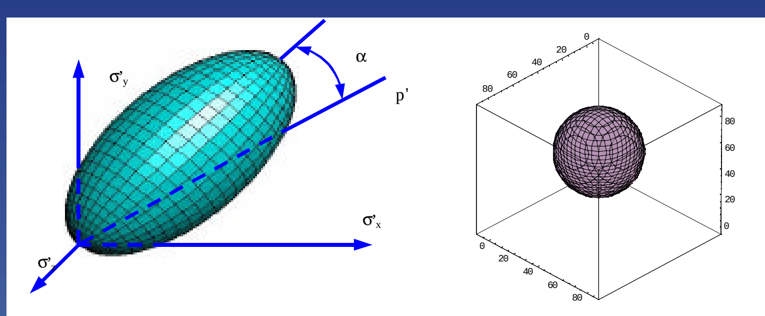


Layout of the presentation

- S-CLAY1 and S-CLAY1S models
- Numerical examples
  - Benchmark Embankment 3
  - Murro Test Embankment
- Conclusions
- References

S-CLAY1 model



$$F = \frac{3}{2}\left[\{\underline{\sigma}_d - p'\underline{\alpha}_d\}^T\{\underline{\sigma}_d - p'\underline{\alpha}_d\}\right] - \left[M^2 - \frac{3}{2}\{\underline{\alpha}_d\}^T\{\underline{\alpha}_d\}\right][p'_m - p']p' = 0$$

Definitions:

Deviatoric stress vector

$$\underline{\sigma}_d = \begin{bmatrix} \sigma'_x - p' \\ \sigma'_y - p' \\ \sigma'_z - p' \\ \sqrt{2}\tau_{xy} \\ \sqrt{2}\tau_{yz} \\ \sqrt{2}\tau_{zx} \end{bmatrix}$$

$$p' = \frac{\sigma'_x + \sigma'_y + \sigma'_z}{3}$$

Deviatoric fabric tensor
(in vector form)

$$\underline{\alpha}_d = \begin{bmatrix} \alpha_x - 1 \\ \alpha_y - 1 \\ \alpha_z - 1 \\ \sqrt{2}\alpha_{xy} \\ \sqrt{2}\alpha_{yz} \\ \sqrt{2}\alpha_{zx} \end{bmatrix}$$

$$\frac{\alpha_x + \alpha_y + \alpha_z}{3} = 1$$

S-CLAY1: Hardening laws

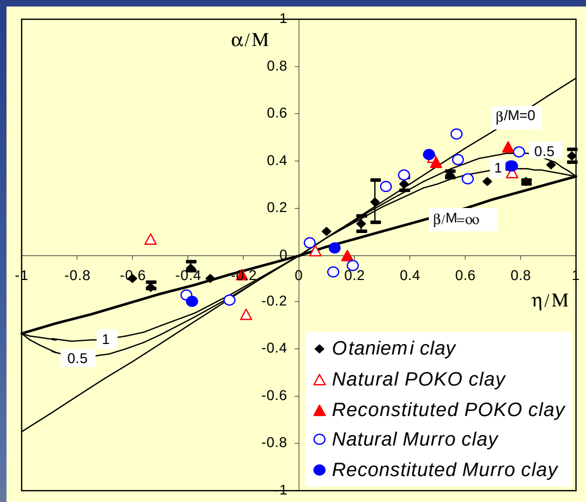
Hardening law controlling the size of the yield surface:

$$dp'_m = \frac{\nu p'_m d\varepsilon^p_v}{\lambda - \kappa}$$

Hardening law controlling the rotation of the yield surface:

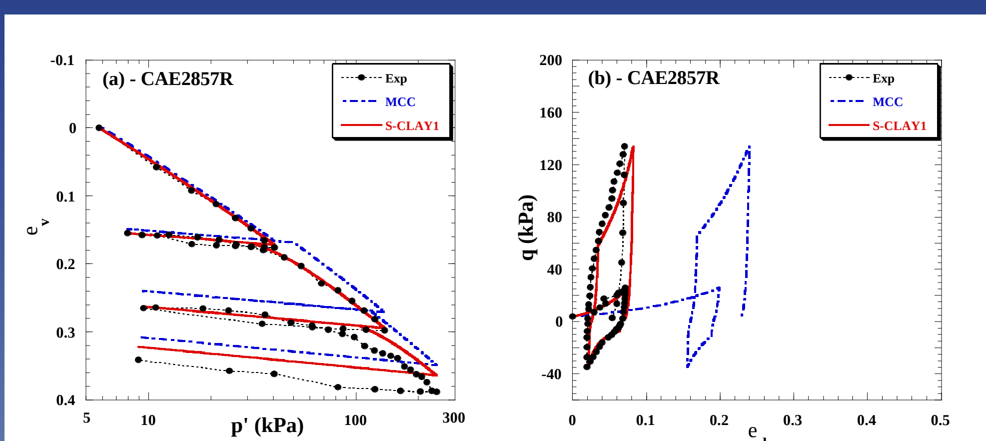
$$d\underline{\alpha}_d = \mu\left[\left(\frac{3\underline{\eta}}{4} - \underline{\alpha}_d\right)\langle d\varepsilon^p_v\rangle + \beta\left(\frac{\underline{\eta}}{3} - \underline{\alpha}_d\right) d\varepsilon^p_d\right] \qquad \underline{\eta} = \frac{\underline{\sigma}_d}{p'}$$

Experimental evidence



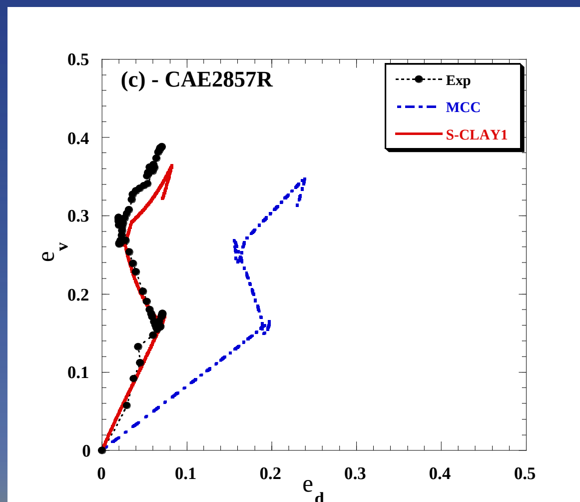
Reconstituted POKO Clay

CAD2857R (η_1=0.65,η_2=-0.22, η_3=0.55)

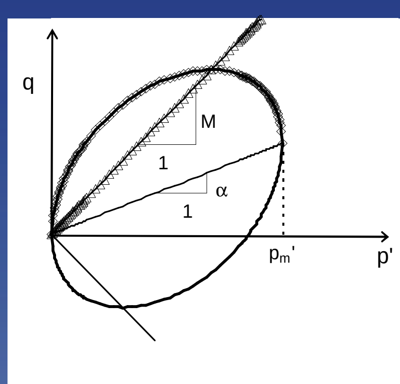


Reconstituted POKO Clay

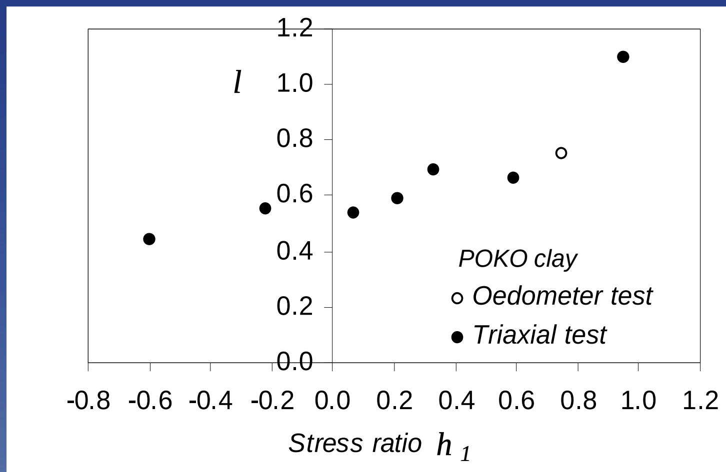
CAD2857R (η_1=0.65,η_2=-0.22, η_3=0.55)



S-CLAY1: Yield surface in triaxial stress space

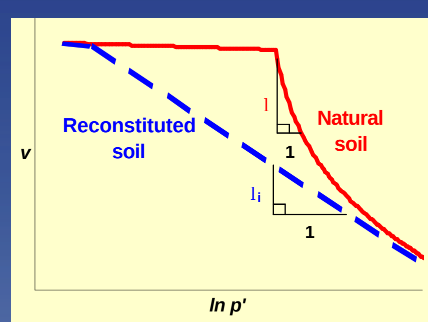


$$F = (q - \alpha p')^2 - (M^2 - \alpha^2)[p'_m - p']p' = 0$$

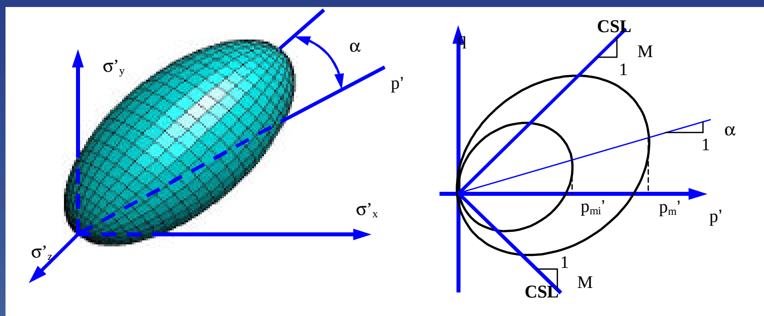


Structure of natural soils

- Soil structure consists of:
  - fabric (***anisotropy***)
  - interparticle **bonding** (*sensitivity*)
- Due to plastic straining gradual degradation of bonding (***destructuration)***



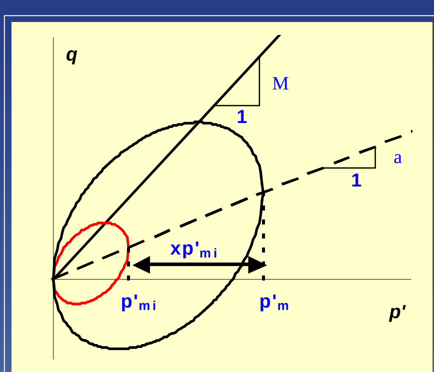
S-CLAY1S models



$$p'_m = (1 + x)\, p'_{mi}$$

$$F = \frac{3}{2}\left[\{\underline{\sigma}_d - p'\underline{\alpha}_d\}^T \{\underline{\sigma}_d - p'\underline{\alpha}_d\}\right] - \left[M^2 - \frac{3}{2}\{\underline{\alpha}_d\}^T\{\underline{\alpha}_d\}\right]\left[p'_m - p'\right]p' = 0$$

S-CLAY1S model



Hardening laws:

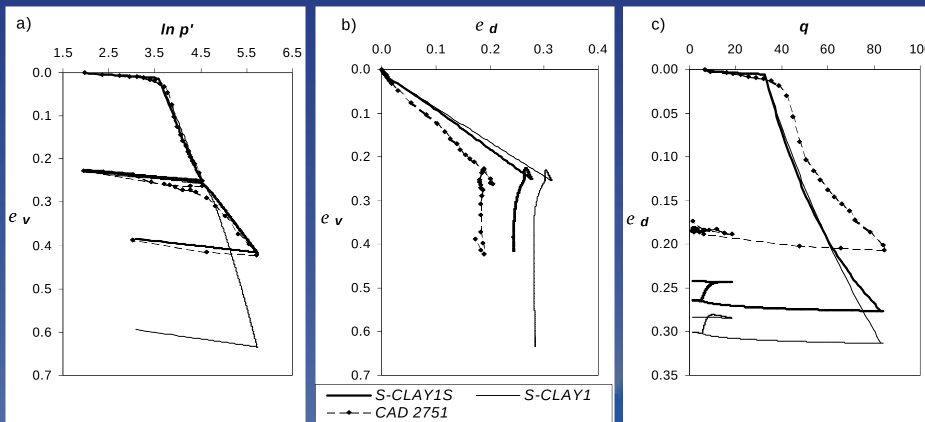
$$dp'_{mi} = \frac{v p'_{mi}}{\lambda_i - \kappa} d\varepsilon^p_v$$

$$d\underline{\alpha}_d = \mu\left[\left(\frac{3\eta}{4} - \underline{\alpha}_d\right)\langle d\varepsilon^p_v\rangle + \beta\left(\frac{\eta}{3} - \underline{\alpha}_d\right)d\varepsilon^p_d\right]$$

Hardening law for destructuration:

$$dx = a\left[(0 - x)\left|d\varepsilon^p_v\right| + b\,(0 - x)\left|d\varepsilon^p_d\right|\right] = -ax\left[\left|d\varepsilon^p_v\right| + b\,\left|d\varepsilon^p_d\right|\right]$$

CAD2751



Determination of additional parameters

Symbol	Definition	Method
α	Initial inclination of the yield curve	Estimated via ϕ'
β	Proportion constant	Estimated via ϕ'
μ	Rate of rotation	$\approx$ (10…20)*λ_{K0}

S-CLAY1

x	Initial amount of bonding	$\approx S_t$ -1
λ_i	Slope on intrinsic compression line	Consolidation test on reconstituted soil
b	Proportion constant	For most clays 0.2
a	Rate of destructuration	Typically 8-10

S-CLAY1S

Fabric tensor for initially cross anisotropic soil

$$\underline{\alpha}_d = \begin{bmatrix} \alpha_x - 1 \\ \alpha_y - 1 \\ \alpha_z - 1 \\ \sqrt{2}\alpha_{xy} \\ \sqrt{2}\alpha_{yz} \\ \sqrt{2}\alpha_{zx} \end{bmatrix} \Rightarrow \underline{\alpha}_d = \begin{bmatrix} \frac{1}{3}(\alpha_x - \alpha_y) \\ -\frac{2}{3}(\alpha_x - \alpha_y) \\ \frac{1}{3}(\alpha_x - \alpha_y) \\ 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \underline{\alpha}_d = \begin{bmatrix} \frac{1}{3}(3\alpha_x - 3) \\ -\frac{2}{3}(3\alpha_x - 3) \\ \frac{1}{3}(3\alpha_x - 3) \\ 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \underline{\alpha}_d = \begin{bmatrix} -\frac{\alpha}{3} \\ \frac{2\alpha}{3} \\ -\frac{\alpha}{3} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\alpha_{xy} = \alpha_{yz} = \alpha_{zx} = 0$$

$$\alpha_x = \alpha_z$$

$$\frac{\alpha_x + \alpha_y + \alpha_z}{3} = 1$$

$$\alpha^2 = \frac{3}{2}\{\underline{\alpha}_d\}^T\{\underline{\alpha}_d\}$$

Implementation of the models

By following the standard procedure

$$[D^{ep}] = [D^e] - \frac{1}{\beta}[D^e]\left(\frac{\partial Q}{\partial \underline{\sigma}'}\right)\left(\frac{\partial F}{\partial \underline{\sigma}'}\right)^T [D^e]$$

with

$$\beta = H + \left(\frac{\partial F}{\partial \underline{\sigma}'}\right)^T [D^e]\left(\frac{\partial Q}{\partial \underline{\sigma}'}\right)$$

For S-CLAY1:

$$H = -\left[\frac{\partial F}{\partial p'_m}\frac{\partial p'_m}{\partial \varepsilon_v^p}\frac{\partial Q}{\partial p'} + \left\{\frac{\partial F}{\partial \underline{\alpha}_d}\right\}^T \left(\left\{\frac{\partial \underline{\alpha}_d}{\partial \varepsilon_v^p}\right\}\left\langle\frac{\partial Q}{\partial p'}\right\rangle + \sqrt{\frac{2}{3}}\left\{\frac{\partial \underline{\alpha}_d}{\partial \varepsilon_d^p}\right\}\sqrt{\left\{\frac{\partial Q}{\partial \underline{\sigma}'_d}\right\}^T\left\{\frac{\partial Q}{\partial \underline{\sigma}'_d}\right\}}\right)\right]$$

For S-CLAY1S:

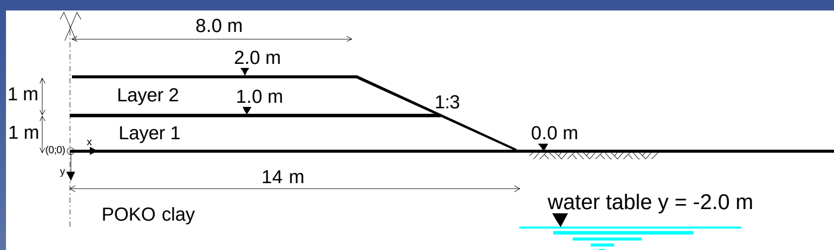
$$H = -\left[\frac{\partial F}{\partial p'_m}\frac{\partial p'_m}{\partial \varepsilon_v^p}\frac{\partial Q}{\partial p'} + \left\{\frac{\partial F}{\partial \alpha_d}\right\}^T\left[\left\{\frac{\partial \alpha_d}{\partial \varepsilon_v^p}\right\}\left\langle\frac{\partial Q}{\partial p'}\right\rangle + \sqrt{\frac{2}{3}}\left\{\frac{\partial \alpha_d}{\partial \varepsilon_d^p}\right\}\sqrt{\left\{\frac{\partial Q}{\partial \sigma'_d}\right\}^T\left\{\frac{\partial Q}{\partial \sigma'_d}\right\}}\right] + \left\{\frac{\partial F}{\partial x}\right\}\left[\frac{\partial x}{\partial \varepsilon_v^p}\left|\frac{\partial Q}{\partial p'}\right| + \sqrt{\frac{2}{3}}\frac{\partial x}{\partial \varepsilon_d^p}\sqrt{\left\{\frac{\partial Q}{\partial \sigma'_d}\right\}^T\left\{\frac{\partial Q}{\partial \sigma'_d}\right\}}\right]\right]$$

To implement the model, the only thing we need is the partial derivatives.

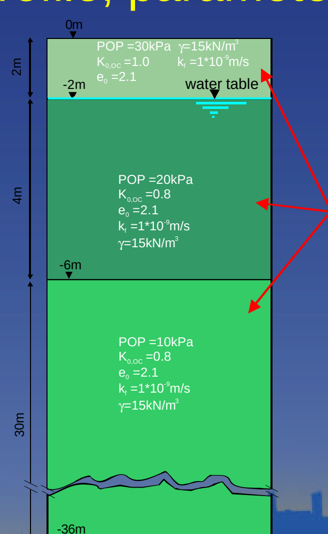
Geometry of the embankment

Embankment parameters:
Model: Mohr Coulomb

E=40000 kN/m^2
ν' =0.3
φ'=38°
c'=1kN/m^2
γ=20kN/m^3



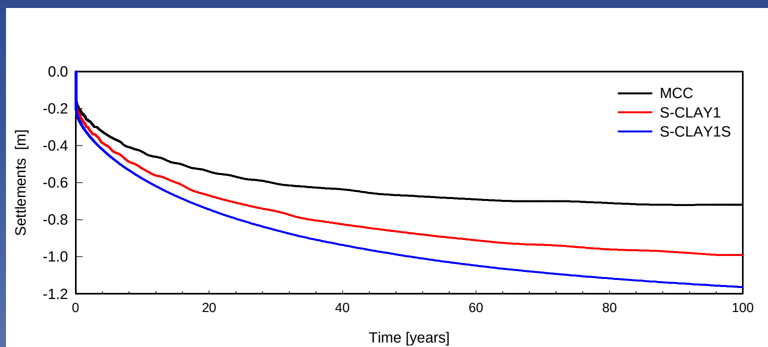
Soil profile, parameter values



	MCC	S-CLAY1	S-CLAY1S
λ (or λ_i)	0.7	0.7	0.26
κ	0.03	0.03	0.03
ν'	0.2	0.2	0.2
M	1.2	1.2	1.2
α_{K_0}	—	0.46	0.46
β	—	0.76	0.76
μ	—	20	20
a	—	—	9
b	—	—	0.2
x_0	—	—	12

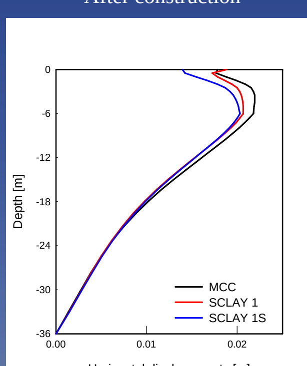
Time settlement curve

Under centreline

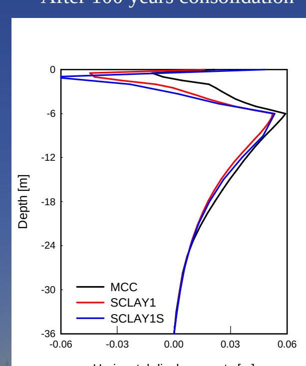


Horizontal displacements

After construction

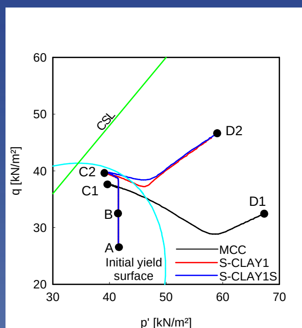


After 100 years consolidation

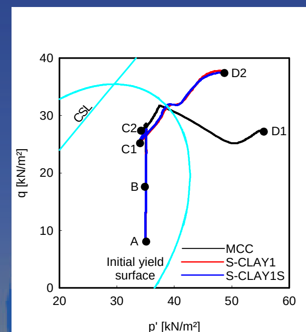


Stress paths on the centreline

4m below ground level



8m below ground level



Murro Test Embankment

- Soil parameters

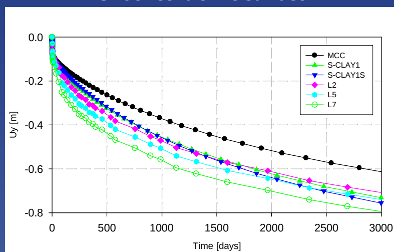
Layer	Depth [m]	γ [kN/m^3]	e_0	POP [kPa]	M	κ	λ	$\lambda_{intrinsic}$	μ	X	a	b	k_x [m/s]	k_y [m/s]
1a	0-1.0	15.8	1.4	20	1.6	0.01	0.16	0.07	45	4	9	0.2	2.47E-09	1.90E-09
1b	1.0-1.6	15.8	1.4	10	1.6	0.01	0.16	0.07	45	4	9	0.2	2.47E-09	1.90E-09
2	1.6-3.0	15.5	1.8	1	1.6	0.03	0.5	0.24	25	8	9	0.2	2.47E-09	1.90E-09
3	3.0-6.7	14.9	2.4	1	1.6	0.036	0.5	0.24	20	8	9	0.2	2.06E-09	1.55E-09
4	6.7-10.0	15.1	2.1	1	1.6	0.03	0.4	0.18	25	10	9	0.2	1.27E-09	1.05E-09
5	10.0-15.0	15.5	1.8	1	1.6	0.034	0.38	0.16	25	9	9	0.2	7.93E-10	6.34E-10
6	15.0-23.0	15.9	1.6	1	1.6	0.034	0.14	0.7	30	9	9	0.2	1.20E-09	9.51E-10

- Embankment parameters

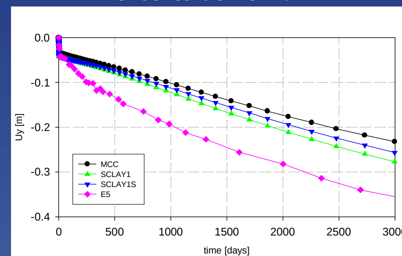
E [MPa]	γ [kN/m^3]	ν	ϕ' [°]	c' [kPa]
40	19.6	0.35	40	2

Time settlement curve

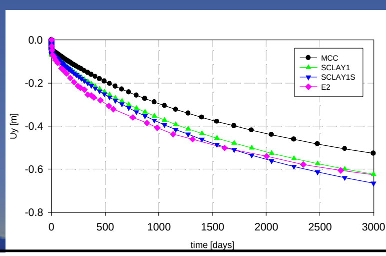
Under centreline surface



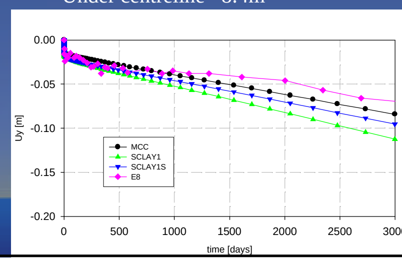
Under centreline –4.4m



Under centreline –1.5m

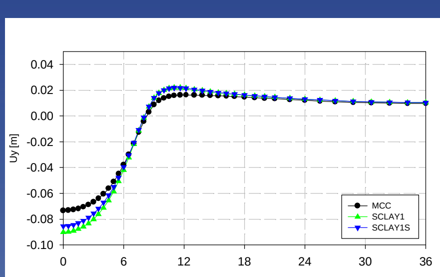


Under centreline –8.4m

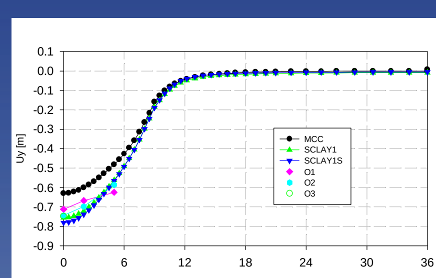


Surface settlements

After construction



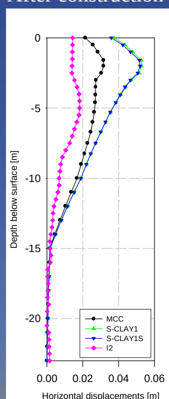
After ~3000 days



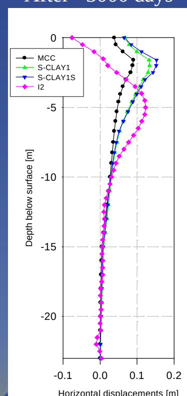
Horizontal displacements

Under the toe of the embankment

After construction

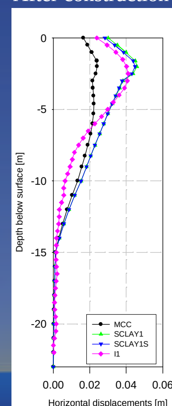


After ~3000 days

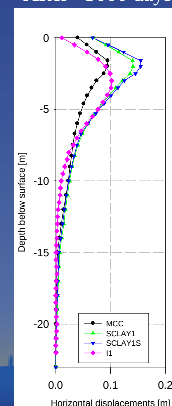


Under the edge of the embankment

After construction

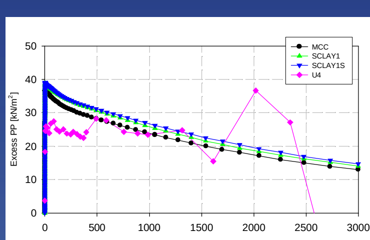


After ~3000 days

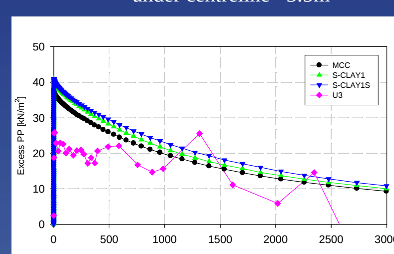


Time versus excess pore pressure

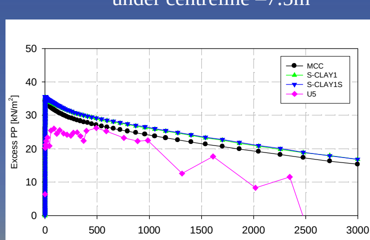
~ under centreline –5.5m



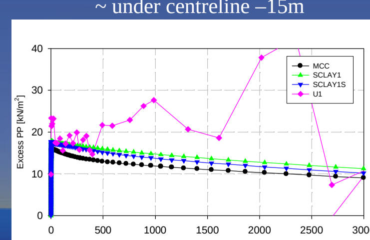
- under centreline –3.5m



~ under centreline –7.5m

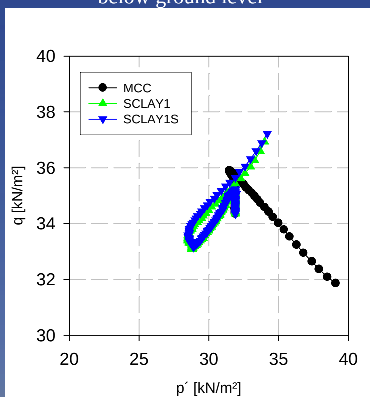


~ under centreline –15m

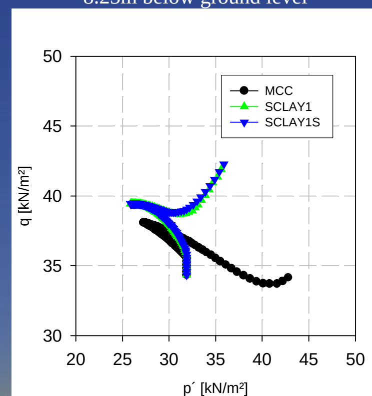


Stress paths

under toe of the embankment 8.25m below ground level



under centreline of the embankment 8.25m below ground level



Conclusions

- Two new constitutive models have been developed: S-CLAY1 (plastic anisotropy) and S-CLAY1S (plastic anisotropy and destructuration)
- Both models have been validated against series of multi-stage triaxial test on both natural and reconstituted soft clays from Finland (Otaniemi, POKO, Murro, Vanttila) and Scotland (Bothkennar)
- Both models have been implemented to SAGE Crisp
- For embankments on soft clays, the predicted vertical settlements increase and the predicted horizontal movements decrease when plastic anisotropy and destructuration are taken into account (compared to the MCC model)

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