BLOCK ANALYSIS METHOD AND ITS APPLICATIONS

M. Gunn and J. Xie

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Faculty of Engineering, Science and Built Environment
London South Bank University

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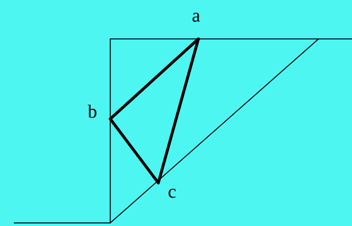
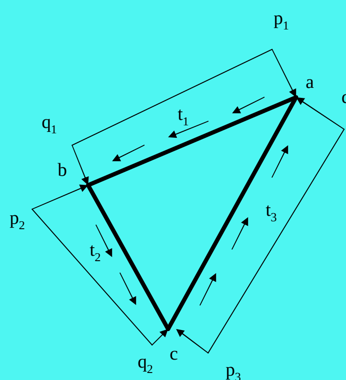
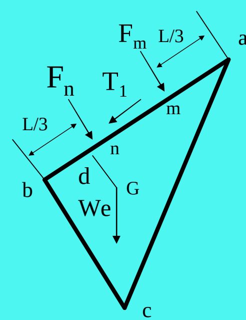
Block element:





Forces on the element



a(x_1, y_1)

b(x_2, y_2)

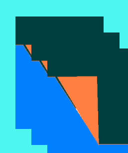
c(x_3, y_3)

G(x_G,y_G)

m(x_m,y_m)

n(x_n,y_n)

Equilibrium equations:



$$F_m = \frac{1}{2} l p_1 \qquad T_1 = t_1 l$$

$$F_n = \frac{1}{2} l q_1 \qquad \textbf{We = S*}\ g$$

$$F_{mx} = \frac{1}{2} p_1 (y_1 - y_2) \qquad F_{my} = \frac{1}{2} p_1 (x_2 - x_1)$$

$$F_{nx} = \frac{1}{2} q_1 (y_1 - y_2) \qquad F_{ny} = \frac{1}{2} q_1 (x_2 - x_1)$$

$$T_{1x} = t_1 (x_2 - x_1) \qquad T_{1y} = t_1 (y_2 - y_1)$$

* S is the area of the element; L is the length of the side “ab”; γ is unit weight of soil.

"ab" side on the element:

$$\left.\begin{aligned} S_{1x} &= F_{mx}+F_{nx}+T_{1x} \\ S_{1y} &= F_{my}+F_{ny}+T_{1y} \\ S_{1m} &= F_{mx}(y_G-y_m)+F_{nx}(y_G-y_n)-F_{my}(x_G-x_m) \\ &\quad -F_{ny}(x_G-x_n)+t_1\cdot l\cdot d \end{aligned}\right\} \quad \begin{bmatrix} S_{1x} \\ S_{1y} \\ S_{1m} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} p_1 \\ q_1 \\ t_1 \end{bmatrix}$$

$$a_{11} = a_{12} = \frac{1}{2}(y_1 - y_2), \qquad a_{13} = x_2 - x_1$$

$$a_{21} = a_{22} = \frac{1}{2}(x_2 - x_1), \qquad a_{23} = y_2 - y_1$$

$$a_{31} = \frac{1}{2}[(y_2 - y_1)(y_G - y_m) + (x_2 - x_1)(x_G - x_m)]$$

$$a_{32} = \frac{1}{2}[(y_2 - y_1)(y_G - y_n) + (x_2 - x_1)(x_G - x_n)]$$

$$a_{33} = -dl$$

For an element:

- $[S_1] = [A_1][R_1]$

$$\begin{bmatrix} A_1 & A_2 & A_3 \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix} + l \begin{bmatrix} 0 \\ -Sg \\ 0 \end{bmatrix} = 0$$

$$[A_e][R_e]+[W_e]\,l = 0$$

For whole structure:

$$[A] = \begin{bmatrix} A_{e1} & 0 & . & . & .0 \\ 0 & A_{e2} & . & . & . \\ . & . & . & . & . \\ . & . & . & . & 0 \\ 0 & . & . & 0 & A_{en} \end{bmatrix}$$

$$[A][R]+[W]\,l = 0$$

$[R] = [R_{e1}, R_{e2}, \ldots, R_{en}]^T$

$[W] = [W_{e1}, W_{e2}, \ldots, W_{en}]^T$

2. Yield condition:

- Tresca material ($\varphi = 0$):
- $C_u \geq t_i \geq -C_u, \ (i=1,2,3)$
  - where C_u is undrained shear strength

3. Linear Programming:

$$\text{Max. } Z = \lambda$$

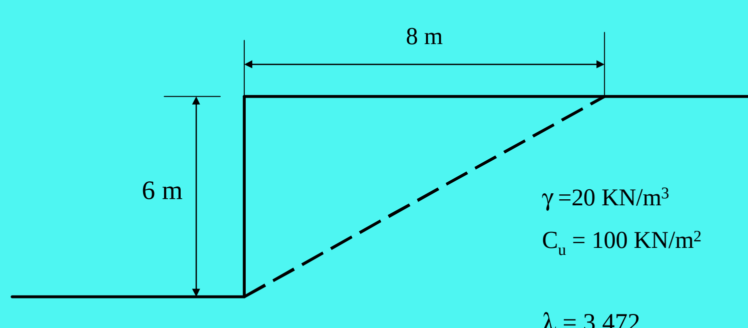
$$\text{Subject to: } AR + W\lambda = 0$$

$$C_u \geq t_i \geq -C_u$$

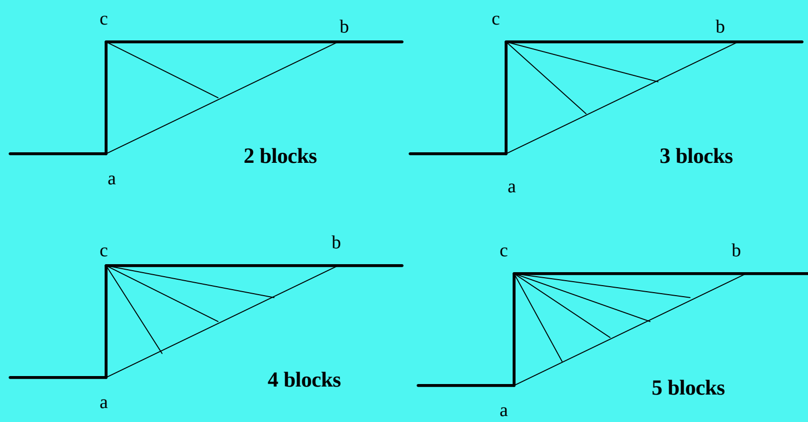
$$\text{all variables } \geq 0$$

Examples:

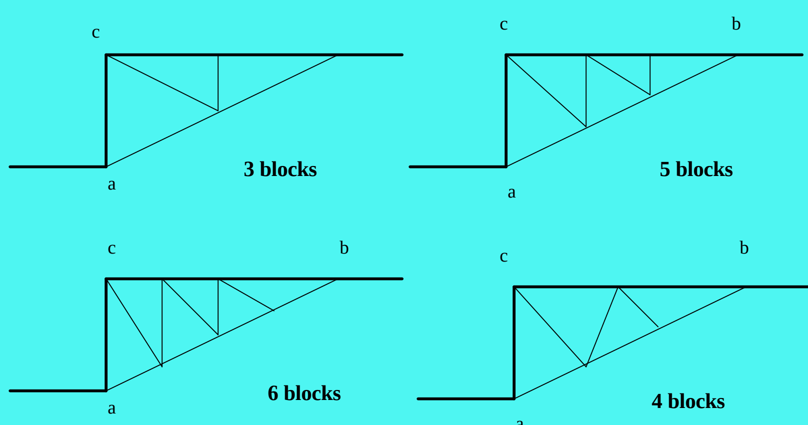
- Vertical cut (1) - straight lines.
  - There are four groups are examined:
    - mother block



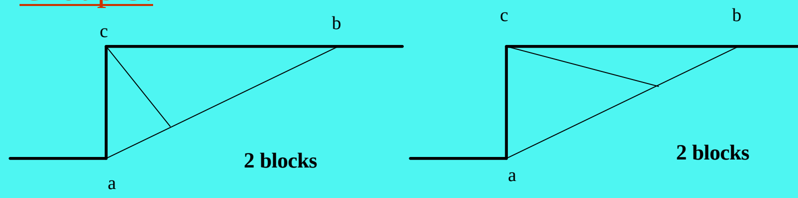
Group 1.



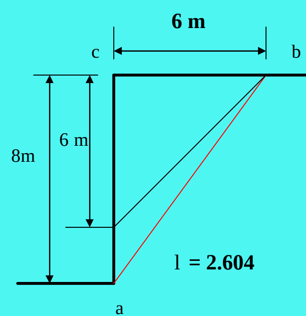
Group 2.

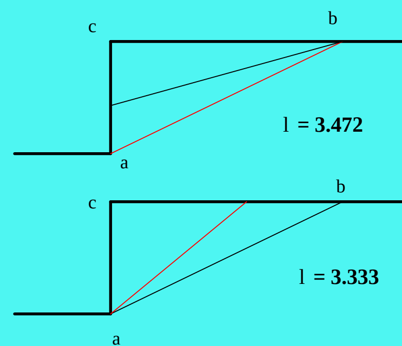


Group 3.



Group 4.



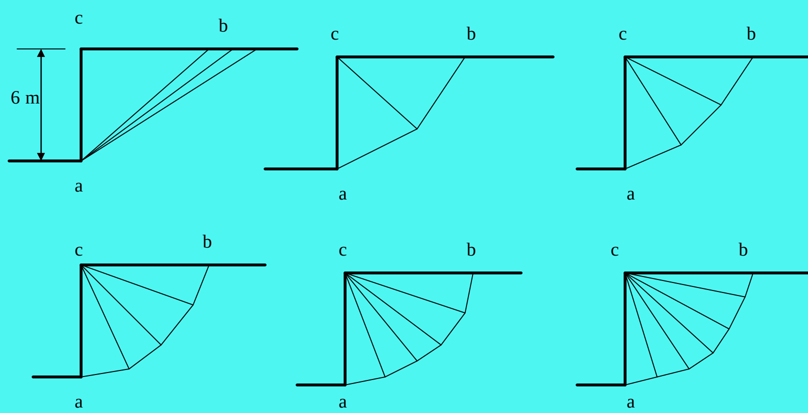


Summary

- 1. Cutting a mother block into smaller son blocks, if all son blocks move together, the load factor is equal to mother block’s.

- 2. If son blocks can move separately, the load factor comes from the collapse slipping side.

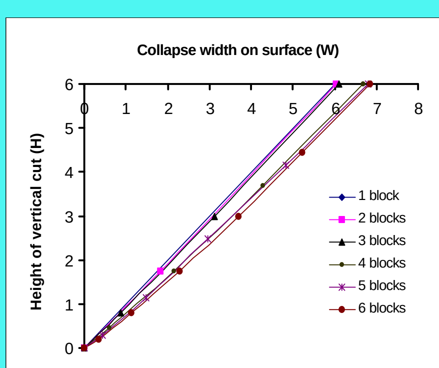
Vertical cut (2): Search optimum slipping lines

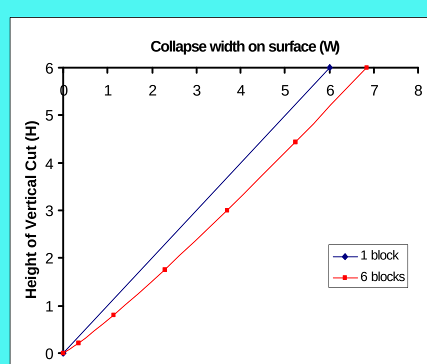


Block Analysis Method:

$$h_c = a * \frac{C_u}{g}$$

Blocks	1	2	3	4	5	6
α	4	3.946	3.9	3.791	3.767	3.655



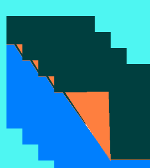


Results:

$$h_c = a * \frac{C_u}{g}$$

Upper bound:	α	Lower bound:	α
Coulomb	4	Drucker et al (1952)	2
Taylar(1948)	3.83	Heyman (1973)	$2\sqrt{2}$
Chen et al (1969)	3.83	Paster (1978)	3.635

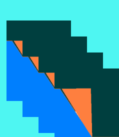
Finite Element method:		Block Analysis method:	
Cascini (1983)	3.764	Authors	3.655



Conclusion

Block Analysis Method can be used in the stability analysis of soil structures.

It has been employed in the stability analysis of vertical cuts and plane strain headings.



It is possible to develop new elements (2D and 3D) and calculate complex stability problems.

Thank you!