New Small Strain Stiffness model for consolidation analyses

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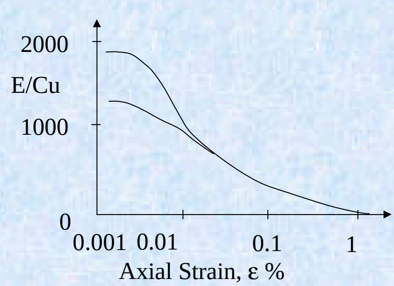
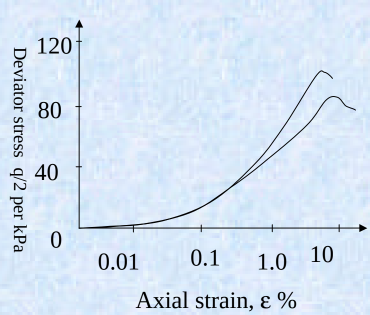
SUMMARY OF PREVIOUS VERSION

Gunn I soil model was restricted to undrained soil response only. Details of this model are given in Ref [1]. The original model uses a power law to control the variation of the undrained bulk modulus with total strain. A Tresca yield criterion is used to control stresses in the plastic zone when the deviatoric stress reaches the shear strength of the soil. The Tresca yield criterion is uses as it is sufficient for undrained response which is restricted to clay materials (ie no angle of friction is used).

Gunn I model, continued

Variation of stiffness and deviatoric stress with strain when suitable laboratory equipments are used for measuring small strain





GUNN II MODEL DESCRIPTION

The new model is aimed for drained, consolidating as well as undrained soil response. It uses a power law to simulate the change of effective soil **bulk modulus**, **and** soil **shear modulus** with total strain. The model also includes a yield criterion to control stresses when the deviatoric stress reaches the shear strength of soil. As the model is suitable for clay as well as sand, the Mohr-Coulomb yield criterion is used for this purpose.

The derived equations are based on soil laboratory measurements of stiffness under a very small axial strain (<0.002%) as reported in Jardine et. al [Ref 2].

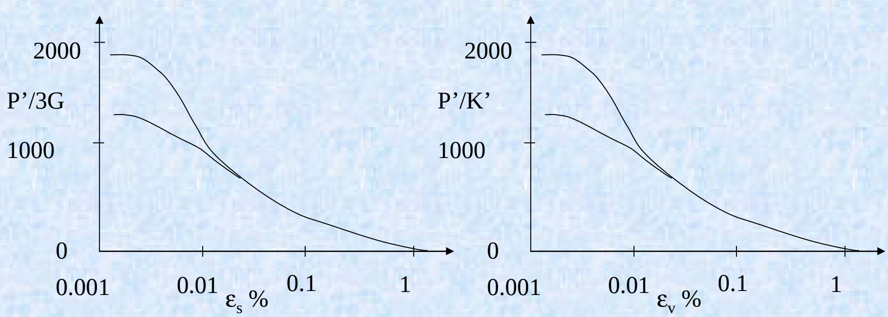


Figure 1 Typical stress-strain graphs showing variation of stiffness during loading

The above stress-strain graphs suggest a relation as follows:

$$q = Ap_o^{'} e_s^{n} \qquad (p' - p_o^{'}) = B.p_o^{'} e_{vol}^{m}$$

The above stress-strain graphs suggest a relation as follows:

where **q** is the deviatoric stress,

p' is the effective mean stress

e_s is the deviatoric strain

e_{vol} is the volumetric strain

A and **B** are modulus parameters (dimensionless)

m and **n** are modulus exponents **p'$_o$** and is the mean effective stress at the start of loading, ie the in-situ mean stress.

Stiffness Formulations

The basic expressions relating the bulk and shear moduli to the total strain are therefore given as follows:

$$\text{tangent}.(3G) = n.A.p_o^{'} e_s^{n-1}$$

$$\text{secant}.(3G) = A.p_o^{'} e_s^{n-1}$$

$$\text{tangent}.(K') = m.B.p_o^{'} e_{vol}^{m-1}$$

$$\text{secant}.(K') = B.p_o^{'} e_{vol}^{m-1}$$

Choice of parameters

The parameters **A**, **n**, **B** and **m** for the model are recovered from the secant Young's modulus measures at two strain levels in an undrained triaxial test

The following parameters are derived according to tests detailed in Hight, et. Al 1993 Ref 6

Shear modulus parameters for Thames Gravel

$$\text{secant}.(3G/p') = 1000 \quad at \quad e_s = 0.0001$$

$$\text{secant}.(3G/p') = 350 \quad at \quad e_s = 0.001$$

$$n = 1 + \frac{\log_{10}\left(\frac{1000}{350}\right)}{\log_{10}\left(\frac{0.0001}{0.001}\right)} = 0.544$$

$$1000 = A \times 0.0001^{-0.456} \qquad giving \;\; A = 15.0$$

Bulk modulus parameters for Thames Gravel

From fig 6b of Ref 6

$$\text{secant}.(K'/p') = 457 \quad at \quad e_{vol} = 10^{-4}$$

$$\text{secant}.(K'/p') = 229 \quad at \quad e_{vol} = 10^{-3}$$

$$m = 1 + \frac{\log_{10}\left(\frac{457}{229}\right)}{-1} = 0.7$$

$$457 = B \times 0.0001^{-0.3} \qquad giving \;\; B = 63.1$$

Parameters for London Clay

A similar procedure is followed for London Clay data, giving

A=2.95

n=0.4

B=2.79

m=0.544

The Gunn II model incorporates a Mohr Coulomb yield surface to allow for plastic yielding when the deviator stress reaches the limit given by the shear strength C and the angle of internal friction ϕ. In addition, the model allows for the variation of the shear strength, **C** ,and stiffness parameter **A** with depth according to the formulae:

$$C = C_o + m_c(y_o - y)$$

$$A = A_o + m_a(y_o - y)$$

where Yo is the elevation at which the undrained Cohesion Co and the parameter ao are measured, mc and ma represent the rate of change of C and a with depth respectively.

Solution procedure in CRISP

The Newton-Raphson method

The Newton-Raphson Method is one of the most widely used iteration method for the solution of the non-linear finite element analysis. It can be written as

$$\mathbf{K}_i^{j-1}\Delta\mathbf{d}_i^j = \mathbf{R}_i - \mathbf{F}_i^{j-1}$$

wherei and j are numbers of the load increment and the iteration respectively,

$\mathbf{K}_i^{j-1}$ is a stiffness matrix based on solutions at the i^{th} load increment and $(j-1)^{th}$ iteration,

$\mathbf{F}_i^0 = \mathbf{R}_{i-1}$

$\mathbf{R}_i - \mathbf{F}_i^{j-1}$ is known as a vector of the out-of-balance (residual) loads due to the non-linearity.

The out-of-balance indicates difference between a correct stress point at the i^{th} load increment and a calculated one. The iteration procedure continues until the out-of-balance load satisfies appropriate convergence criteria. This scheme is also referred to as the fully Newton-Raphson Method.

Stress point algorithms

The determination of the out-of-balance load is a key issue in the Newton-Raphson method. It is based on the stress point, which adds the latest stress changes to that at the beginning of the iteration. There are several algorithms to evaluate the stress point.

At the i^{th} load increment

Step 1: Assemble stiffness using tangential D matrix

$$\mathbf{K}_i^{j-1}\Delta\mathbf{d}_i^j = \mathbf{R}_i - \mathbf{F}_i^{j-1}$$

Step 2: Evaluate incremental strain

$$\Delta e = \mathbf{B}\Delta\mathbf{d}_\mathbf{i}^\mathbf{j}$$

Step3: Update total strain

$$e_i^j = e_i^{j-1} + \Delta e$$

Step4: evaluate volumetric and deviatoric strains

Step5: evaluate tangential stiffness using tangential values of K' and G

Step 6: evaluate incremental stress using tangential stiffness

$$\Delta s = D_{\text{tangent}}\Delta e$$

Step 7: Update stress

$$s_i^j = s_i^{j-1} + \Delta s$$

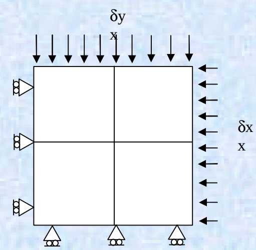
Step 8: Check if yield has taken place. If so, adjust stresses using Mohr Coulomb yield surface.

If material is elastic use **secant stiffness** and update stresses again

$$s_i^j = s_0 + D_{\text{secant}} e_i^j$$

Test 1 (Non-linear elastic only)

This consists of a plane strain problem with four elements. The material is assumed to be weightless. The elements are subjected to a horizontal strain of 0.01 and a vertical displacement of 0.015. This is applied in one load block of 10 increment, then removed in another load block of 10 increments



Parameter description	Parameter (units)	value
Gunn II shear modulus parameter	**A**	2.95
Gunn II shear modulus exponent	**n**	0.4
Gunn II bulk modulus parameter	**B**	2.79
Gunn II bulk modulus exponent	**m**	0.544
Shear Strength	**C**	0
Angle of internal friction	**Phi**	0
Water bulk modulus	K_w (kN/m^3)	0
Bulk unit weight of soil	g_b (**kN/m^3**)	0
Coefficient of permeability	**kx**	0
Coefficient of permeability	**ky**	0
Poisson’s ratio	**-**	0
evmin	e_{vol_min}	0.00001
evmax	e_{vol_max}	10
edmin	e_{dev_min}	0.00001
edmax	e_{dev_max}	10
Flag, 1 for non-linear elastic, 0 for Mohr Coloumb yield surface. Default is 0	ielastic	0

Results

The effective mean stress against increments is shown below. This shows that effective mean stress increases up to 206 kN/m2 at the end of the loading block (increment 10), then reduces symmetrically to zero at the end of the unload block (increment 20).

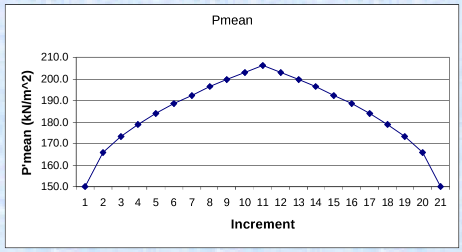


Figure 3 Graph of effective mean stress against increments for Gunn II non-linear elastic response, displacement controlled loading.

The deviatoric stress against increments is shown below. This shows that deviatoric stress increases up to 67.1 kN/m2 at the end of the loading block (increment 10), then redeuces symmetrically to zero at the end of the unload block (increment 20).

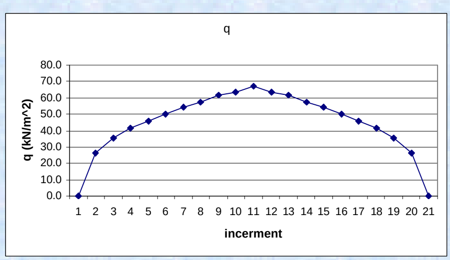
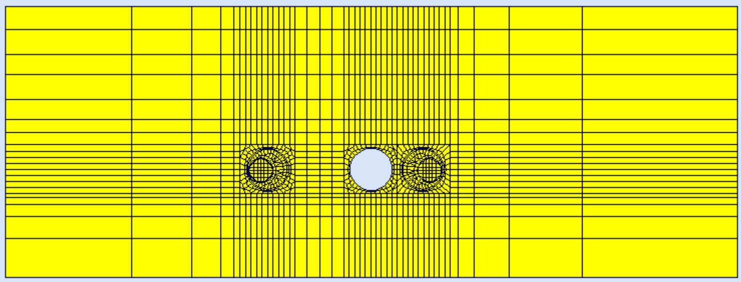


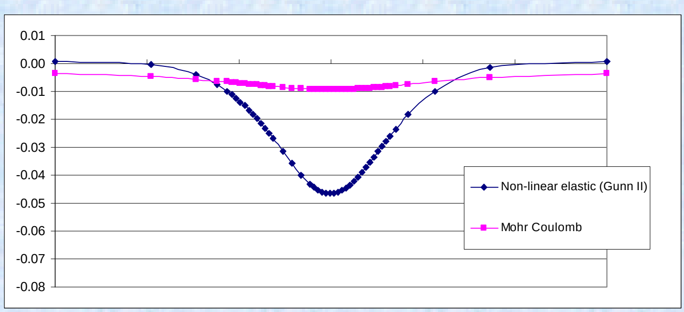
Figure 4 Graph of deviatoric stress against increments for Gunn II non-linear elastic response, displacement controlled loading.

Example of ground surface prediction for tunnel excavation using Gunn II non-linear elastic model and Mohr Coulomb elastic plastic model with typical parameters for London Clay



Fe mesh of tunnel excavation. Developed by Tube Lines engineers (P. Wright and T. Morrison)

Ground surface profiles after excavation of central tunnel showing results for Gunn II non-linear elastic model and Mohr Coulomb model



Current Difficulties:

The Gunn II model often produces K’ and G values which result in a negative poisson’s ratio. This does not appear to be a problem when using the non-linear elastic option with no yield surface. But when the Mohr Coulomb yield surface is invoked in this model, convergence problems occur due to negative poisson’s ratio. Substituting a typical value for poisson’s ratio does not solve the convergence problem. Therefore, further effort is needed to identify the reasons why the calculated K’ and G are incompatible for acceptable poisson’s ratio.

REFERENCES

Ref 1 Gunn, M.J. (1992) Proceedings of the Wroth Memorial Symposium, St Catherine's College, Oxford, 27-29 July 1992 Edited by Houlsby and Schofield, Published by Thomas Telford London

Ref 2 Jardine, R.J.,Symes, M.J, and Burland, J.B.(1984). The measurement of soil stiffness in triaxial apparatus, Geotechnique, vol. XXXIV, No. 3, pp.323-340

Ref 3 Jardine, R.J., Potts, D.M., Fourie, A.B. and Burland, J.B. (1986) Studies of the influence of non-linear stress-strain characteristics in soil-structure interaction, Geotechnique, Vol. XXXVI, No. 3, pp.377-396

Ref 4 Naylor, D.J, Pande G.N., Simpson, B and Tabb, R. (1981). Finite Elements in Geotechnical Engineering, Pineridge Press, Swansea

Ref 5 Duncan, J.M and Chang, C.Y (1969) Non-Linear analysis of stress and strain in soils. J. Soil Mech. Found. Div., Proc. ASCE, vol.96,pp1629-1653

Ref 6 Hight, et al 1993